

6.5 - Properties of Logarithms

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Warmup

Expand the expression

1. $\log_3(xy)$

$$\log_3 x + \log_3 y$$

2. $\log_4(rst)$

$$\log_4 r + \log_4 s + \log_4 t$$

3. $\log_2\left(\frac{x}{r}\right)$

$$\log_2 x - \log_2 r$$

4. $\log_b\left(\frac{\sqrt{x}}{p}\right)$

$$\frac{1}{2} \log_b x - \log_b p$$

5. $\log_3(5\sqrt[3]{a})$

$$\log_3 5 + \frac{1}{3} \log_3 a$$

6. $\log_{10}(ac)^2$

$$2 \log_{10} a + 2 \log_{10} c$$

6.4 - Transformations of Exponential and Logarithmic Functions

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Transformations

$$f(x) = e^x$$

How do you transform $f(x)$ to obtain $h(x)$?

$$a. h(x) = e^{x+2}$$

Translate left 2

$$b. h(x) = -e^{x-4}$$

Translate right 4

Reflect across x-axis

$$c. h(x) = \left(\frac{1}{e^{3x}}\right) + 7$$

Reflect y-axis
horz. shrink 1/3
translate up 7

$$d. h(x) = 2 - \frac{1}{e^{x/2}}$$

Horz. stretch 2
Reflect y-axis, x-axis
Translate up 2

6.5 - Properties of Logarithms

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Properties of Logarithms

$$\log_3 x^5 = 5 \log_3 x$$

$$4 \log_5 x - \log_5 4 = \log_5 x^4 - \log_5 4$$

$$\log_3 xy = \log_3 x + \log_3 y$$

$$= \log_5 \frac{x^4}{4}$$

$$\log_7 \frac{x}{y} = \log_7 x - \log_7 y$$

Condense to one log

1. $\log_b 6 + \log_b 5 - \log_b 2$

$$\log_b 15$$

2. $\frac{1}{2} \log_b r + \frac{1}{2} \log_b s$

$$\log_b \sqrt{rs}$$

3. $\log_2 A + 1 + 2 \log_4 B$

$$\log_2 2AB = \log_4 4A^2B^2$$

6.5 - Properties of Logarithms

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Logarithmic Equations

$$\log(x + 2) + \log(x - 1) = \log 4$$

$$\log(x + 2)(x - 1) = \log 4$$

$$(x + 2)(x - 1) = 4$$

$$x^2 + x - 2 = 4$$

$$x^2 + x - 6 = 0$$

$$(x + 3)(x - 2) = 0$$

$$x = -\cancel{3}, 2$$

6.6 - Solving Exponential and Logarithmic Equations

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Exponential Equations

$$3^{2x} - 2 \cdot 3^x - 8 = 0$$

$$z = 3^x$$

$$z^2 - 2z - 8 = 0$$

$$(z - 4)(z + 2) = 0$$

$$z = 4, \cancel{-2}$$

$$4 = 3^x$$

$$\log_3 4 = \log_3 3^x$$

$$\log_3 4 = x$$

$$\frac{\log 4}{\log 3} = x$$

$$x = 1.26\dots$$

Practice

1. $3^{2x} - 5 \cdot 3^x + 4 = 0$

$0, \log_3 4$

2. $e^{2x} - e^x - 6 = 0$

$\ln 3$

3. $5^{2x+1} + 3 \cdot 5^x - 2 = 0$

$\log_5 \frac{2}{5}$

6.6 - Solving Exponential and Logarithmic Equations

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Exponential Equations

$$3^{2x} - 2 \cdot 3^x - 8 = 0$$

$$z = 3^x$$

$$z^2 - 2z - 8 = 0$$

$$(z - 4)(z + 2) = 0$$

$$z = 4, -2$$

$$4 = 3^x$$

$$\log_3 4 = \log_3 3^x$$

$$\log_3 4 = x$$

$$\frac{\log 4}{\log 3} = x$$

$$x = 1.26\dots$$

Practice

1. $3^{2x+1} - 4 \cdot 3^x - 4 = 0$

$$\log_3 2$$

2. $5^{2x-1} - 2 \cdot 5^x + 5 = 0$

$$1$$

3. $5^{2x+2} - 3 \cdot 5^{x+1} + 2 = 0$

$$\log_5 \frac{2}{5}, -1$$

6.6 - Solving Exponential and Logarithmic Equations

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Exponential Equations

$$e^x + e^{-x} = 5$$

$$e^x(e^x + e^{-x} = 5)$$

$$e^{2x} + 1 = 5e^x$$

$$e^{2x} - 5e^x + 1 = 0$$

$$e^x = \frac{5 \pm \sqrt{21}}{2} = 0.2087121525$$
$$= 4.7912878475$$

$$x = -1.5667992371$$

$$x = 1.566799237$$

Practice

1. $e^x - 6e^{-x} = 1$

$$\ln 3 = 1.10$$

2) $\frac{e^x + e^{-x}}{e^x - e^{-x}} = 3$

$$\frac{1}{2} \ln 2 = 0.35$$

6.6 - Solving Exponential and Logarithmic Equations

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Exponential Equations

$$e^x + e^{-x} = 5$$

$$e^x(e^x + e^{-x} = 5)$$

$$e^{2x} + 1 = 5e^x$$

$$e^{2x} - 5e^x + 1 = 0$$

$$e^x = \frac{5 \pm \sqrt{21}}{2} = 0.2087121525$$
$$= 4.7912878475$$

$$x = -1.5667992371$$

$$x = 1.566799237$$

Practice

1. $3^{-2x} - 4 \cdot 3^{-x} + 3 = 0$

$$x = 0, -1$$

2) $5^{6x} - 2 \cdot 5^{3x} - 3 = 0$

$$\frac{1}{3} \log_5 3$$

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Logarithm Product Property

$$\log_3 xy = \log_3 x + \log_3 y$$

$$\begin{aligned}\log_7 \frac{x}{y} &= \log_7(x \cdot y^{-1}) \\ &= \log_7 x + \log_7 y^{-1} \\ &= \log_7 x - \log_7 y\end{aligned}$$

Practice

1. $\log \frac{x^2 + 3x + 2}{x^3 - 1}$

2. $\log \frac{x^{-1} - 9}{x^{-2} - 4}$

$$\log(x + 1) + \log(x + 2) - \log(x - 1) - \log(x^2 + x + 1)$$

$$\log x + \log(1 - 9x) - \log(1 - 2x) - \log(1 + 2x)$$

6.5 - Properties of Logarithms

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Properties of Logarithms

$$3^{\log_3 x} = 3x - 4$$

$$\log_3(3^{\log_3 x}) = \log_3(3x - 4)$$

$$\log_3 x = \log_3(3x - 4)$$

$$x = 3x - 4$$

$$x = 2$$

Practice

$$\left(\frac{2}{\sqrt[3]{4}}\right)^{\log_2(x+2)} = 8$$

$$x = 510$$

